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Mesons in gauge/gravity dual with large number of fundamental fields

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Abstract: In view of extending gauge/gravity dualities with flavour beyond the probe approximation, we establish the gravity dual description of mesons for a three-dimensional super Yang-Mills theory with fundamental matter. For this purpose we consider the fully backreacted D2/D6 brane solution of Cherkis and Hashimoto in an approximation due to Pelc and Siebelink. The low-energy field theory is the IR fixed point theory of three-dimensional $\mathcal{N} = 4$ SU(N_c) super Yang-Mills with N_f fundamental fields, which we consider in a large N_c and N_f limit with N_f/N_c finite and fixed. We discuss the dictionary between meson-like operators and supergravity fluctuations in the corresponding near-horizon geometry. In particular, we find that the mesons are dual to the low-energy limit of closed string states. In analogy to computations of glueball mass spectra, we calculate the mass of the lowest-lying meson and find that it depends linearly on the quark mass.

Keywords: Field Theories in Lower Dimensions, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The AdS/CFT correspondence may be generalized such as to describe degrees of freedom in the fundamental representation of the gauge group. This was first suggested in [1] where a D7 brane probe is embedded in the AdS/CFT D3 brane background. Fundamental degrees of freedom are given by open strings stretching between the D3 branes and the D7 brane probe. These are dual to open strings on the D7 probe wrapping $AdS_5 \times S^3$, as given by the Dirac-Born-Infeld action for the D7 brane probe in the low energy limit. The associated field theory is a $\mathcal{N} = 2$ supersymmetric Yang-Mills theory in $d = 4$ with N_7 fundamental hypermultiplets, where N_7 is the number of D7 branes.

The meson spectrum for the $\mathcal{N} = 2$ supersymmetric theory of [1] is calculated in [2]. The meson masses are shown to scale linearly with the quark mass as required by supersymmetry. An essential ingredient in this calculation is to consider the limit $N_7 \ll N_3$ with N_7 fixed, such that the backreaction of the D7 probe on the geometry can be neglected. A similar calculation of the meson spectrum for the $\mathcal{N} = 1$ supersymmetric Klebanov-Strassler background with an embedded D7 brane probe is performed in [3].

In [4], the D7 brane probe is embedded into the non-supersymmetric Constable-Myers background [5].[1] In this way a gravity dual description of spontaneous chiral symmetry breaking is found, including a Goldstone boson which was noted to have properties of the

[1] See also [6] for further results on the probe configuration in this background.

η' meson: The η' is a Goldstone boson of U(1) only in the limit $N \to \infty$. This is just the limit in which the supergravity approximation is valid. Moreover in [4] it is shown that the pseudo-Goldstone boson mass scales with the square root of the quark mass in agreement with field theory expectations. In [7], a similar computation is performed within type IIA supergravity for the D4 brane black hole metric of [8] with a D6 brane probe. This background metric has the advantage of being regular in the interior (i.e. in the infrared of the field theory). Spontaneous chiral symmetry breaking occurs in this background as well. Moreover, a holographic version of the Vafa-Witten theorem is derived in this brane set-up [7]. — A D5 brane probe in the Maldacena-Nuñez background is investigated in [9] (see also [10]).

All of these calculations are performed in the probe approximation, neglecting the backreaction of the probe brane. In view of further progress it is necessary to go beyond both the probe limit and the supergravity approximation. A step in this direction was taken in [11], where $1/N$ corrections to the supergravity limit were considered in order to derive the Witten-Veneziano formula for the η' meson in the D4/D6 probe configuration. Further related results are also found in [12]. In [13] a probe D7 brane is embedded in the backreacted D3/D7 solution, and the spectrum of scalar fluctuations is computed to first order in N_f.

In this paper we take a step towards describing a large number of fundamentals by computing the meson spectrum in a full supergravity metric for intersecting branes without embedding a probe brane. To this effect, we consider the type IIA D2/D6 brane system for which the supergravity metric is known [14] for arbitrary N_2 and N_6, the number of D2 and D6 branes, respectively. The low energy dynamics of this system is described by a 2+1 dimensional $\mathcal{N} = 4$ SU(N_2) super Yang-Mills theory which is coupled to an adjoint and N_6 fundamental hypermultiplets. The theory, whose gauge coupling is dimensionful, flows to a superconformal fixed point in the IR.

The D2/D6 configuration is T-dual to both the D3/D5 configuration considered in [15, 16] and to the D3/D7 configuration of [1]. Recently, the backreaction of D7 branes in the $AdS_5 \times S^5$ and $AdS_5 \times T^{1,1}$ geometries were studied in [13]. However, as compared to the backreacted D3/D7 configuration, which leads to a 3+1 dimensional theory with flavour, the D2/D6 system has the essential technical advantage that the backreaction of the D6 branes does not lead to a deficit angle. The field theory corresponding to D3/D7 is not conformal if N_f/N_c is kept finite and fixed. In contrast, the three-dimensional theory corresponding to D2/D6, which is superrenormalizable, flows to a non-trivial IR fixed point even for N_f/N_c finite.

The solution for the fully localized D2/D6 brane intersection was given by Cherkis and Hashimoto in [14]. We consider this solution in the limit $1 \ll N_6 \ll N_2$ with N_6/N_2 fixed such that the backreaction of the D6 branes must be taken into account. This geometry has the important feature that the fluctuations of the D6 branes induce fluctuations of the supergravity background. The corresponding near-horizon geometry describes the full renormalization group flow of the 2+1 dimensional super Yang-Mills theory from the UV to the interacting fixed point in the IR. For the purpose of computing the meson spectrum, it is enough to focus on the supergravity description of the IR fixed point theory which is given by the metric of Pelc and Siebelink [17]. This metric approximates the region close to the center of the Cherkis-Hashimoto background corresponding to the IR regime of the field theory.

Near the boundary, the Pelc-Siebelink metric reduces to $AdS_4 \times S^7/\mathbb{Z}_{N_6}$. However, the interior of this geometry is deformed due to a non-vanishing quark mass. The corresponding Kaluza-Klein spectrum has been worked out by Entin and Gomis [18]. Moreover, these authors relate some of the supergravity excitations to chiral operators in the 2+1-dimensional theory. Chiral operators belong to the short representations of the superconformal algebra and as such they are protected against quantum corrections. For this reason these operators remain unrenormalized in the IR fixed point theory, where they are dual to the supergravity fluctuations on $AdS_4 \times S^7/\mathbb{Z}_{N_6}$. We extend this analysis to meson-like operators involving components of the fundamental superfields in view of computing the meson spectrum.

Using mirror symmetry we argue that the field theory operators involving fundamentals are dual to supergravity modes, ie. to the low energy limit of closed strings. This is in contrast to the probe approximation, where the meson-like operators are dual to open strings on the probe brane. Another meson/closed string duality has been proposed in [12], in which D3-branes are considered on an orbifold of the type $\mathbb{C}^3/(\mathbb{Z}_3 \times \mathbb{Z}_3)$. There the lightest pseudo-scalar meson is conjectured to be dual to a particular twisted RR field which is generated by closed strings.

The conjectured meson/closed string duality allows us to compute the meson spectrum by a supergravity computation in the Pelc-Siebelink background. Similarly as in calculations of the glueball mass spectrum [19]–[22], we use a plane wave ansatz for the fluctuations of the scalar field dual to the mesons. We numerically solve the resulting equation by using standard shooting techniques. The quark mass enters the Pelc-Siebelink metric as a free parameter. Varying this parameter, we determine the mass of the lowest-lying meson in the spectrum and its dependence of the quark mass. We find a linear behaviour on the quark mass as expected for supersymmetric field theories.

The outline of the paper is as follows. In section 2 we present the supergravity setup for the D2/D6 brane configuration, and discuss the $2+1$-dimensional $\mathcal{N}=4$ supersymmetric gauge theory associated with this background. In section 3 we provide the gauge/gravity dictionary for this set-up. The main feature is that the field theory operators involving fundamental fields are dual to closed strings in the Pelc-Siebelink geometry. These closed strings are generated by the fluctuations of the D6 branes. We also calculate the lowest-lying meson mass for the gauge theory from supergravity and find it to scale linearly with the quark mass, in agreement with field theory expectations.

2. Holography of the D2/D6 brane intersection

2.1 Configuration

We consider N_2 D2-branes with world-volume coordinates along x^0, x^1, x^2 parallel to N_6 D6-branes along $x^0, ..., x^6$. This brane intersection preserves 8 supercharges and is T-dual to the orthogonal D3/Dp brane intersections ($p \in \{3,5,7\}$) whose holography is studied in [1, 15, 23, 24].

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There are several kinds of strings in the D2/D6 set-up. First, there are open strings ending on the D2-branes (2-2 strings) and open strings stretching between the D2- and the D6-branes (2-6 and 6-2 strings). Closed strings decouple upon taking the usual $l_s \to 0$ limit. We also send $g_s \to 0$ such that the 2+1-dimensional gauge coupling

$$g^2_{\mathrm{YM2}} = g_s l_s^{-1} \tag{2.1}$$

remains fixed. In order to keep the 't Hooft coupling λ_2 constant, we take the limit

$$N_2, N_6 \to \infty\,, \qquad g^2_{\mathrm{YM2}} \to 0\,, \qquad \lambda_2 \equiv g^2_{\mathrm{YM2}} N_2 = \mathrm{const}\,, \qquad \nu \equiv N_6/N_2 = \mathrm{const.} \tag{2.2}$$

The 6+1-dimensional 't Hooft coupling

$$\lambda_6 \equiv g^2_{\mathrm{YM6}} N_6 = (2\pi)^4 g_s l_s^3 N_6 = (2\pi l_s)^4 g^2_{\mathrm{YM2}} N_6 = (2\pi l_s)^4 \lambda_2 \nu \tag{2.3}$$

vanishes in this limit implying the decoupling of open 6-6 strings. The massless open string degrees of freedom of the D2/D6 intersection correspond to a $\mathcal{N} = 4, d = 3$ super Yang-Mills multiplet coupled to N_6 hypermultiplets in the fundamental representation and to one hypermultiplet in the adjoint representation of the gauge group. The D2/D6 intersection and the decoupling of the strings is shown on the left hand side of figure 1.

The full supergravity solution for the D2/D6 intersection including the backreaction of the D6-branes has been found by Cherkis and Hashimoto [14]. Note that in the limit (2.2), the backreaction of the D6-branes on the geometry is not negligible as can be seen by comparing the tension of both stacks of branes,

$$T_{\mathrm{D6}} = \frac{\nu}{(2\pi l_s)^4} T_{\mathrm{D2}}\,, \tag{2.4}$$

which is very large at low energies ($l_s \to 0$) if ν is fixed.

The near-horizon limit of the Cherkis-Hashimoto metric can be obtained upon taking the coupling $g^2_{\mathrm{YM2}} \to \infty$. The D2-branes are then completely resolved by their near-horizon geometry, whereas the D6-branes remain in the background. A singularity in the metric signals the existence of open strings ending on the D6-branes [17]. At strong coupling, we thus have closed string modes arising from both the D2 and the D6-branes and open strings attached to the D6-branes. Figure 1 shows the relevant strings in the near-horizon geometry.[2]

A summary of the string dualities in the D2/D6 brane configuration is as follows.[3] In the limit given by (2.2), we have two kinds of open strings on the gauge theory side of the correspondence: open strings stretching between the D2 and D6-branes (2-6 strings), and open strings ending on the D2-branes (2-2 strings). In the low-energy theory these strings give rise to composite operators of purely adjoint fields and to meson-like operators involving fundamental matter fields. Each of these operators maps unambiguously to a

[2]To be more precise, figure 1 shows the strings in the corresponding *ten-dimensional* near-horizon geometry which follows by dimensional reduction.

[3]A detailed discussion of the fluctuation-operator map is given in section 3.1 below.

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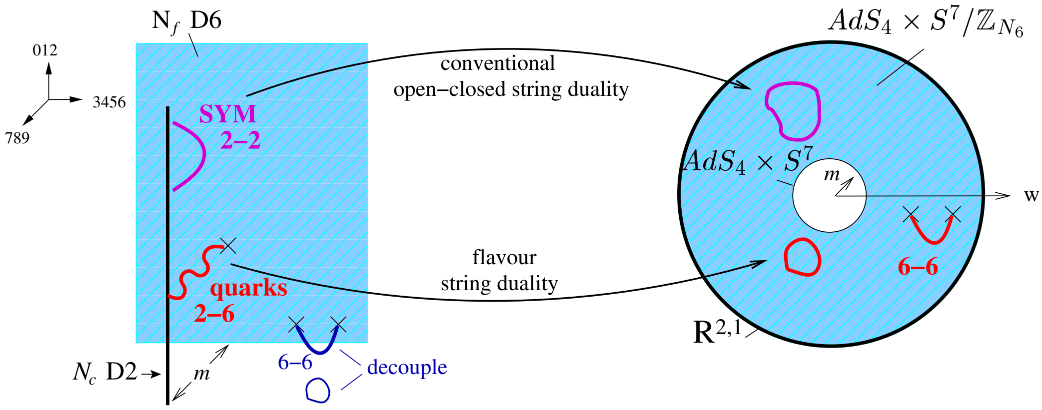

Figure 1: Two-fold open-closed string duality: (a) standard duality between adjoint operators (2-2 strings) and a class of closed strings. (b) flavour string duality between meson operators (2-6 strings) and another class of closed strings. This will be discussed in detail in section 3.1.

supergravity fluctuation generated by closed strings. This splits the closed string modes into two classes: Those which are dual to purely adjoint operators and those dual to meson operators. As shown in figure 1, we conjecture the AdS/CFT duality to act 'twice' here: First, we have the standard AdS/CFT duality between open 2-2 strings and a class of closed strings in type IIA string theory. Secondly, we have a further duality between open 2-6 strings and another class of closed strings. In the low-energy limit, the two classes of closed strings lead to different supergravity multiplets. This flavour open-closed string duality will be relevant for the computation of the meson spectrum.

2.2 Field theory

In the following we discuss the world-volume theory of the D2/D6 intersection, a $\mathcal{N} = 4$ SU(N_c) super Yang-Mills theory ($N_c \equiv N_2$) which is coupled to an adjoint and $N_f \equiv N_6$ fundamental hypermultiplets. The action can be conveniently written in $\mathcal{N} = 2$, $d = 3$ superspace formalism and is derived in appendix A. The theory flows to a superconformal fixed point in the IR [25]. It belongs to a class of three-dimensional super Yang-Mills theories studied in a number of papers [26]–[32].

First, let us consider the system in the absence of D6-branes. The low-energy effective action on the D2-branes is 2+1-dimensional $\mathcal{N} = 8$ SYM theory with gauge group SU(N_c) which is generated by the dynamics of 2-2 strings. The theory has a dimensionful gauge coupling and is not conformal in the UV. The gauge coupling is determined by the type IIA string coupling which blows up in the IR. At strong coupling, it is more appropriate to consider this theory as the world-volume theory of M2-branes instead of D2-branes whose near-horizon geometry is $AdS_4 \times S^7$. This agrees with the behaviour of the field theory which flows to a superconformal fixed point in the IR [25].

By adding N_6 D6-branes, we break half of the supersymmetry such that the theory on the intersection becomes a 2+1-dimensional $\mathcal{N} = 4$ gauge theory. Its field content is as follows. The $\mathcal{N} = 8$ vector multiplet splits into a $\mathcal{N} = 4$ vector and a $\mathcal{N} = 4$

N=4	N=2	components	spin	j_Φ	(j_H, j_V)	$\mathrm{U}(N_f)$	Δ
		σ, ϕ_3	0	0	$(0,1)$	1	1
vector	Φ_3, Σ	$\lambda_3, \bar{\lambda}$	$\frac{1}{2}$	0	$(\frac{1}{2}, \frac{1}{2})$	1	$\frac{3}{2}$
		A_i	1	0	$(0,0)$	1	1
adj.		$\phi_1,\ \phi_2$	0	$\frac{1}{2}$	$(\frac{1}{2}, 0)$	1	$\frac{1}{2}$
hyper	Φ_1, Φ_2	$\lambda_1, \bar{\lambda}_2$	$\frac{1}{2}$	$\frac{1}{2}$	$(0, \frac{1}{2})$	1	1
fund.		$q_f,\ \tilde{q}_f$	0	0	$(\frac{1}{2}, 0)$	N_f	$\frac{1}{2}$
hyper	$Q_f, \tilde{Q}_f$	$\psi_f, \tilde{\psi}_f$	$\frac{1}{2}$	0	$(0, \frac{1}{2})$	N_f	1

Table 1: Field content of the D2/D6 intersection.

adjoint hypermultiplet or, equivalently, into three $\mathcal{N} = 2$ chiral superfields $\Phi_1, \Phi_2,\ \Phi_3$, and a $\mathcal{N} = 2$ vector multiplet V. In addition we have $N_f \equiv N_6$ $\mathcal{N} = 4$ fundamental hypermultiplets ("quarks") generated by the low-energy dynamics of 2-6 strings. These hypermultiplets consist of $\mathcal{N} = 2$ chiral superfields Q_f and $\tilde{Q}_f$ with the flavour index running from 1 to N_f. The theory has thus a global $\mathrm{U}(N_f)$ flavour symmetry. The decoupling of the 6-6 strings guarantees that the the $\mathrm{U}(N_f)$ symmetry on the world-volume of the D6-branes acts as a global symmetry rather than a gauge symmetry. Note also that due to the indistinguishability of left- and right-handed fermions, there is no chiral symmetry $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$ in three dimensions.

$\mathcal{N} = 4$ gauge theories in three dimensions have an OSp(4/4) supergroup. Its even part is the product of the conformal group SO(3,2) and an $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ R-symmetry group. The $\mathrm{SU}(2)_V$ symmetry is the enhanced $\mathrm{U}(1)_R$ symmetry of $\mathcal{N} = 2, d = 4$ supersymmetry upon reduction to three dimensions and is realized in the dual string theory as rotations in the 789 directions.[4] As a subgroup of an anomaly-free group, the $\mathrm{U}(1) \subset \mathrm{SU}(2)_V$ is non-anomalous, unlike $\mathrm{U}(1)_R$ in four dimensions. The $\mathrm{SO}(4) \approx \mathrm{SU}(2)_H \times \mathrm{SU}(2)_\Phi$ isometry in the 3456 directions consists of a global $\mathrm{SU}(2)_\Phi$ symmetry rotating Φ_1 and Φ_2 and the $\mathrm{SU}(2)_H$ part of the $\mathcal{N} = 4$ R-symmetry.

Table 1 summarizes the R-charges and engineering dimensions of the fields of the D2/D6 intersection. The dimension Δ and the R-symmetry quantum numbers j_V and j_H of the scalars saturate the inequality $\Delta \geq j_V + j_H$ [33]. For this reason, the conformal dimensions of scalars and spinors in the vector multiplet differ from those of scalars and spinors in the hypermultiplets. In components the theory contains seven adjoint scalars $\phi_{1,2,3}$, σ and four adjoint spinors $\lambda_{1,2,3}$, λ (gaugino). There is also the gauge field A_i which can be explicitly dualized to a scalar only in the abelian case. The components of the fundamental hypermultiplets are the scalars q_f, $\tilde{q}_f$ and the spinors ψ_f, $\tilde{\psi}_f$.

2.2.1 Mass terms and explicit flavour symmetry breaking on the Higgs branch

Adding mass terms for the fundamental fields corresponds to separating the D2-branes from the D6-branes in the transverse directions x^7, x^8, x^9. These directions are parametrized by the expectation values of the complex scalar ϕ_3 and the real scalar σ. These are the lowest

[4]For a detailed discussion on the R-symmetry enhancment see [28].

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components of the scalar and vector multiplets Φ_3 and V, respectively. The 2-6 strings give rise to quarks with a (tree-level) mass $m = \sqrt{\langle\sigma\rangle^2 + |\langle\phi_3\rangle|^2}$ via the interaction terms $\langle\sigma\rangle\bar{\psi}\psi$ and $\langle\phi_3\rangle\tilde{\psi}\psi$. In contrast to the four-dimensional field theory corresponding to the D3/D7 system, the U(1) group inside the global symmetry group U(N_f) is a symmetry for any choice of quark masses [34].[5] This can also be seen in the D-brane set-up, where the U(1) group is a subgroup of SU$(2)_V$ which rotates the directions x^7, x^8, x^9. A vacuum expectation value for the vector $X_V = (\sigma, \mathrm{Re}\,\phi_3, \mathrm{Im}\,\phi_3)$ is invariant under rotations on a two-plane transverse to the vector X_V. This corresponds to the U(1) subgroup.

In a three-dimensional field theory in which all quarks have equal masses, parity is broken. There is however a way to preserve parity: The D6-branes have to be devided into two stacks of $N_6/2$ D6-branes, where one stack is located a distance $-m$, the other one a distance $+m$ away from the D2-branes. Then the quarks come in pairs with equal magnitude but opposite sign leading to a parity-preserving field theory [34, 27].[6]

In theories with an even number of flavours, such mass terms break the flavour group U(N_f) down to U($N_f/2$)$\times$U($N_f/2$) explicitly. The Higgs branch of the field theory considered here is parametrized by $(N_f/2)^2$ mesons $M^i_j = Q^i\tilde{Q}_j$, where Q_i, $\tilde{Q}_j$ are the fundamental chiral multiplets with indices $i, j = 1, \ldots, N_f/2$ [27]. These mesons are elements of the coset space U(N_f)/(U($N_f/2$) $\times$ U($N_f/2$)). More on the Higgs as well as on the Coulomb branches can be found in [27].

2.3 D2/D6 background and its near-horizon limit

In the previous section we have discussed the D2/D6 brane intersection in terms of its world-volume field theory. We now study the dual supergravity description and review some properties of the corresponding near-horizon geometry.

Cherkis and Hashimoto derive the D2/D6 brane solution [14] by lifting the D2/D6 system to M-theory, where the D2-branes become M2-branes and the D6-branes turn into a Taub-NUT space. After solving a harmonic equation, the geometry is reduced back to ten dimensions. This approach guarantees the preservation of 8 supercharges.

The Cherkis-Hashimoto background encaptures the full renormalization group flow of the 2+1 dimensional field theory to the fixed point in the IR. In the following we focus on the simpler supergravity solution found earlier by Pelc and Siebelink [17]. This solution is dual only to the superconformal IR fixed point theory.

Pelc and Siebelink exploit the fact that close to the center of the Taub-NUT space, the geometry is approximated by the orbifold $\mathbb{C}^2/\mathbb{Z}_{N_6}$. The D2/D6 system lifts to M2-branes at $\mathbb{C}^2/\mathbb{Z}_{N_6}$, as opposed to M2-branes at a Taub-NUT space (as in the Cherkis-Hashimoto background). The corresponding eleven-dimensional near-horizon metric is

[5] In the four-dimensional theory associated with the D3/D7 system [1] massive quarks explicitly break the chiral U(1) symmetry.

[6] Such a brane set-up lifts to a stack of N_2 M2-branes at a double Taub-NUT space (DTN) which is a multi-Taub-NUT space with two centers corresponding to two stacks of D6-branes. The core of the DTN geometry (corresponding to the IR regime in the field theory) is approximated by the Eguchi-Hanson metric. A numerical supergravity solution corresponding to M2-branes in an Eguchi-Hanson space has recently been found in [35].

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given by [17]

$$ds^2 = \frac{w^2}{16(LF)^{2/3}}dx_{||}^2 + (LF)^{1/3}\left(\frac{dw^2}{w^2} + d\tilde{\Omega}_7^2\right), \tag{2.5}$$

with

$$d\tilde{\Omega}_7^2 = d\beta^2 + \cos^2\beta d\Omega_3^2 + \sin^2\beta d\tilde{\Omega}_3^2 \tag{2.6}$$

and $d\tilde{\Omega}_3$ the metric on $S^3/\mathbb{Z}_{N_6}$,

$$d\tilde{\Omega}_3^2 = \frac{1}{4}d\Omega_2^2 + \left[\frac{1}{N_6}d\psi + \frac{1}{2}(1-\cos\theta)d\varphi\right]^2. \tag{2.7}$$

Here w parametrizes a radial direction and $0 \leq \beta \leq \frac{\pi}{2}$, $0 \leq \theta \leq \pi$, $0 \leq \varphi, \psi \leq 2\pi$ are angular coordinates. For massless quarks the factor $F = 1$ and the metric is $AdS_4 \times S^7/\mathbb{Z}_{N_6}$ with the scale $L = l_p^6\pi^2 N_2 N_6/2$.

For large N_2 this background is a reliable classical geometric description with a small curvature. As discussed in [17] the geometry is eleven-dimensional as long as $N_6 \ll N_2^{1/5}$ (or, equivalently, $\nu \ll N_2^{-4/5}$). This implies that ν must be very small but non-vanishing in the large N_2 limit. As reviewed in appendix B of this paper, for $0 \ll \nu \ll 1$ the appropriate description is given in terms of its ten-dimensional reduced geometry. For $\nu \gg 1$ the ten-dimensional curvature becomes large and the geometric description breaks down. As discussed in detail in [17], the field theory becomes weakly coupled at large ν due to a simplification of the Feynman diagrams.[7]

Massive quarks are obtained by separating the D2 from the D6-branes by a distance m as shown on the left hand side of figure 1. This distance determines the bare quark mass $\frac{m}{2\pi l_s^2}$ in units of the string tension. In the eleven-dimensional picture m is the distance of the M2-branes from the orbifold singularity which are located at

$$\beta = \frac{\pi}{2}, \qquad \theta = 0, \qquad w = m, \qquad \psi = 0 \bmod 2\pi. \tag{2.8}$$

The separation of the branes affects only the function F which is a measure for the back-reaction of the D6-branes. It is given by [17]

$$F(w;m) = \frac{1}{N_6}\sum_{k=1}^{N_6}\left(\frac{w_k}{w}\right)^{-3}, \quad \frac{w_k}{w} = 1 + \frac{m}{w} - 2\sqrt{\frac{m}{w}}\sin\beta\cos\frac{\theta}{2}\cos\left(\frac{\psi - 2\pi k}{N_6}\right). \tag{2.9}$$

Plots of $F(w;m)$ are shown in figure 2 for different values of $y = \sin\beta\cos\frac{\theta}{2}$. For fixed angles ψ, θ and β, the function $F(w;m)$ typically starts from zero at $w = 0$, increases to a maximum at $w \approx m$ and decreases to one at $w \to \infty$. Asymptotically, i.e. for radii $w \gg m$ the background is $AdS_4 \times S^7/\mathbb{Z}_{N_6}$, which corresponds to the fact that the mass of the quarks is irrelevant at high energies. The backreaction of the D6-branes is small at the point where F diverges.[8] At this point the Pelc-Siebelink background approaches the near-horizon geometry of the D2-branes which is $AdS_4 \times S^7$.

[7] The Feynman diagrams are planar and have only quark loops, i.e. they do not contain loops of adjoint fields.

[8] There is however no curvature singularity at this point, since $R \sim 1/F^{1/3}$.

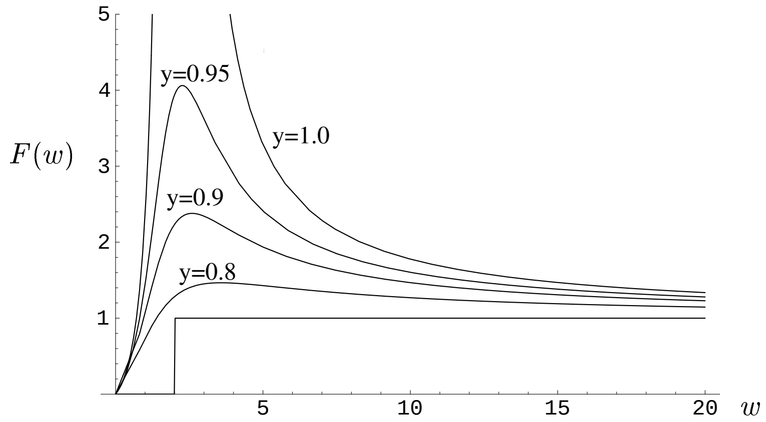

Figure 2: Plot of $F(w; m = 2)$ for different values of $y = \sin\beta\cos\frac{\theta}{2}$ $(\psi = 0)$. The step function $\Theta(w - m)$ is plotted just to guide the eye.

3. Fluctuation-operator map and meson spectrum

In this section we calculate the lowest-lying meson mass for the D2/D6 configuration. In contrast to calculations in the probe approximation, the supergravity degrees of freedom dual to meson operators are already included in the D2/D6 background. We find the meson mass as a function of the quark mass by simply solving the appropriate supergravity wave equations in the Pelc-Siebelink geometry. Since the equations are highly non-linear, the use of numerics is necessary.

This approach is analogous to computations of the glueball mass spectrum in confining gauge/gravity duals. However, we use a different field-operator dictionary which maps meson-like operators to corresponding supergravity fluctuations. In contrast to similar computations in the probe approximation, we do not need a Dirac-Born-Infeld action.

3.1 Fluctuation-operator map

In the following we discuss the dictionary between supergravity fluctuations on $AdS_4 \times S^7/\mathbb{Z}_{N_6}$ and conformal operators in the three-dimensional field theory. In particular, we focus on meson-like operators and their dual supergravity scalars.

In the field theory we have two kinds of chiral operators: First, there are composite operators which consist of purely adjoint fields. An example for such an operator is $\mathrm{Tr}\,\phi_{i_1}\ldots\phi_{i_k}$ $(i_j = 1, 2, 3)$ which also occurs in the $\mathcal{N} = 8$ SYM theory. Secondly, there are meson-like operators which can be considered as a chain of adjoint fields descending from the $\mathcal{N} = 4$ vector multiplet with fundamental matter degrees of freedom (e.g. quarks) at both ends. The operator $\bar{q}X_V^A q$ is an example for a meson-like operator which is bilinear in fundamental scalars q. It is also possible to construct an operator with the same quantum numbers as this operator by simply replacing the fundamental fields by fields of the adjoint hypermultiplet. For example, the operators

$$\bar{q}X_V^A q \qquad \text{and} \qquad \mathrm{Tr}\,\bar{\phi}X_V^A\phi \qquad (\phi = \phi_1, \phi_2) \tag{3.1}$$

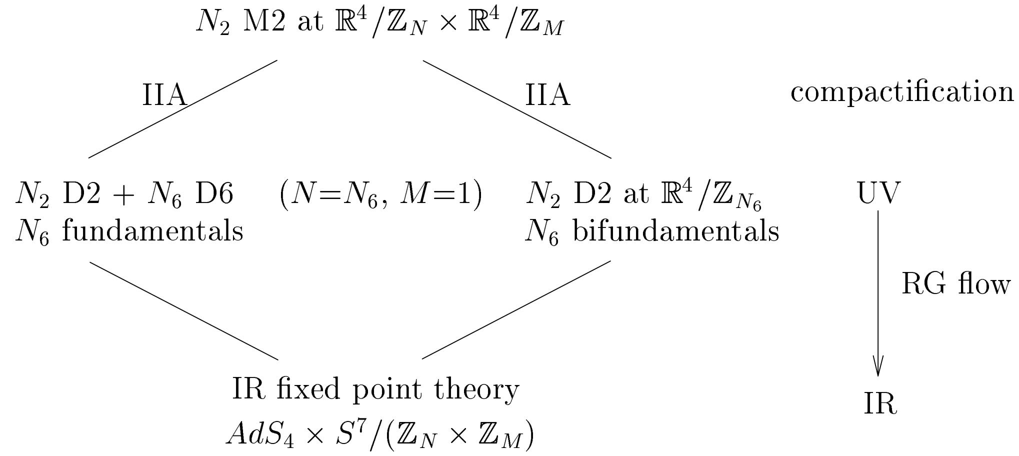

Figure 3: Mirror symmetry.

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both transform in the $(\mathbf{3},\mathbf{1})$ of $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ and have the same engineering dimension. This is due to the same scaling behaviour of the fields in the adjoint and fundamental hypermultiplets. In contrast, the engineering dimension of fields in the vector multiplet differs by 1/2 from that of fields with same spin in the hypermultiplets.

According to the AdS/CFT prescription, chiral operators with given quantum numbers belong to the short representations of the supersymmetry group and couple to corresponding supergravity fluctuations. The spectrum of supergravity excitations on $AdS_4 \times S^7/\mathbb{Z}_{N_6}$ is worked out in [18]. The scalar spectrum is given by the Kaluza-Klein harmonics on $AdS_4 \times S^7$ with mass squared $m^2 = \frac{1}{4}k(k-6)$ $(k \geq 2)$, upon projecting out degrees of freedom which are not invariant under the orbifold group. The resulting supergravity spectrum separates into two classes of fluctuations corresponding to different representations of the global symmetries: One class is dual to adjoint operators, while the other class contains fluctuations dual to meson-like operators. The origin of fluctuations dual to operators with fundamentals is due to the orbifold projection. This is remarkable since without projection, all fluctuations are dual to operators with fields in the adjoint representation only. The orbifold projection ensures the presence of fluctuations dual to meson operators.

One way to understand how the orbifold projection yields fluctuations dual to meson operators with fields in the fundamental representation is given by mirror symmetry [29]. Mirror symmetry relates two compactifications of M-theory on $\mathbb{R}^4/\mathbb{Z}_N \times \mathbb{R}^4/\mathbb{Z}_M$ [25]. Considering N_2 M2-branes near the orbifold $\mathbb{R}^4/\mathbb{Z}_N \times \mathbb{R}^4/\mathbb{Z}_M$, type IIA string theory may be obtained either by compactifying on $\mathbb{R}^4/\mathbb{Z}_N$ or on $\mathbb{R}^4/\mathbb{Z}_M$. The first compactifaction leads to the intersection of N_2 D2-branes and N D6-branes with the orbifold $\mathbb{R}^4/\mathbb{Z}_M$ placed transverse to the D2-branes but on the world-volume of the D6-branes. The corresponding world-volume field theory is $\mathrm{SU}(N_2)^M$ super Yang-Mills theory with N fundamental hypermultiplets and M bifundamental hypermultiplets. The second compactification leads to the same theory with M and N interchanged. In other words, mirror symmetry interchanges the fundamentals with the bifundamentals. In the IR both theories flow to the same fixed point theory which is dual to supergravity on $AdS_4 \times S^7/(\mathbb{Z}_N \times \mathbb{Z}_M)$.

We are interested in the case $N = N_6$ and $M = 1$ which is shown in figure 3. Compactifying a direction transverse to the orbifold, we obtain N_2 D2-branes at the orbifold $\mathbb{R}^4/\mathbb{Z}_{N_6}$. This is the compactification on the right hand side of figure 3. The corresponding field theory of this set-up is a quiver gauge theory with N_6 bifundamentals which follows by a projection of the D2-brane theory. On the gravity side the fluctuation spectrum is obtained by an orbifold projection from the Kaluza-Klein spectrum on $AdS_4 \times S^7$. Since on both sides of the correspondence the spectrum is obtained by an orbifold projection, there must be some supergravity fluctuations which are dual to operators involving bifundamentals. Now compactification on the orbifold circle of $\mathbb{R}^4/\mathbb{Z}_{N_6}$ gives the D2/D6 brane intersection which is the mirror dual theory shown on the left hand side of figure 3. Since mirror symmetry maps the bifundamentals to fundamentals in the mirror dual theory, we expect the existence of fluctuations on $AdS_4 \times S^7/\mathbb{Z}_{N_6}$ which are dual to operators involving fundamentals in the D2/D6 intersection.

Such supergravity fluctuations can indeed be found in the D2/D6 set-up. Let us consider two fluctuations corresponding to $k = 2$ and $k = 4$ in the scalar family which are both tachyons with mass squared $m^2 = -2$. These tachyons do not lead to an instability since their mass lies above the Breitenlohner-Freedman bound $m^2_{\mathrm{BF}} = -\frac{9}{4}$. One of them transforms in the $(\mathbf{1}, \mathbf{3})$ of $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ and is dual to a $\Delta = 1$ operator. The other transforms oppositely in the $(\mathbf{3}, \mathbf{1})$ and is dual to a $\Delta = 2$ operator.[9]

The first scalar couples to a dimension $\Delta = 1$ superconformal primary operator in the IR fixed point theory. Chiral operators are in the short representations of the superconformal algebra and as such, their dimension is protected from quantum corrections. Therefore it is possible to express the operator in terms of fields in the short distance theory. The chiral primary is the sum of a bilinear in the scalars $q^\alpha_f = (q_f, \tilde{q}_f)$ of the fundamental hypermultiplets $(Q_f, \tilde{Q}_f)$ and a bilinear in the scalars $\phi^\alpha = (\phi^1, \phi^2)$ of the adjoint hypermultiplet (Φ_1, Φ_2). It is given by [18]

$$\mathcal{C}^I = \sum_{f=1}^{N_6} \bar{q}^\alpha_f \sigma^I_{\alpha\beta} q^\beta_f + \mathrm{Tr}\, \bar{\phi}^\alpha \sigma^I_{\alpha\beta} \phi^\beta \qquad (I = 1, 2, 3)\,, \tag{3.2}$$

where $\sigma^I_{\alpha\beta}$ is an element of $\mathrm{SU}(2)_H$ such that the operator transfroms in the $(\mathbf{1}, \mathbf{3})$ of the R-symmetry group $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$.

The second scalar is dual to the $\Delta = 2$ operator $(A = 1, 2, 3)$

$$\mathcal{O}^A = \sum_{f=1}^{N_6} \left(\bar{\psi}^i_f \sigma^A_{ij} \psi^j_f + 2 \bar{q}^\alpha_f X^{Aa}_V T^a q^\alpha_f \right) + \mathrm{Tr}\, \bar{\lambda}^i \sigma^A_{ij} \lambda^j + \mathrm{Tr}\, 2 \bar{\phi}^\alpha X^{Aa}_V T^a \phi^\alpha \tag{3.3}$$

transforming as $(\mathbf{3}, \mathbf{1})$. Here X^A_V are the three scalars transverse to the D6-branes while $\psi^i = (\psi, \tilde{\psi})$ and $\lambda^i = (\lambda_1, \lambda_2)$ are the spinorial components of $Q, \tilde{Q}$ and Φ_1, Φ_2, respectively.

Both operators $\mathcal{C}^I$ and $\mathcal{O}^A$ are unambiguously fixed by the R-symmetry quantum numbers, the conformal dimension and gauge invariance. It is not possible to construct an operator out of fields in the adjoint vector multiplet which has the same quantum numbers.

[9]The mass of a p-form on an AdS_{d+1} space is related to the dimension Δ of a $(d-p)$-form operator by $m^2 = (\Delta - p)(\Delta + p - d)$. Here $\Delta = \frac{k}{2}$ and $p = 0$.

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This can easily be shown for the $\Delta = 1$ operator $\mathcal{C}^I$. None of the $\Delta = 1$ fields in table 1 meets the required R-symmetry quantum numbers $(\mathbf{1}, \mathbf{3})$ such that $\mathcal{C}^I$ must be a bilinear in the $\Delta = 1/2$ scalars q_f, $\tilde{q}_f$, q_1 and q_2. The operator $\mathcal{C}^I$ is the sum of all possible combinations of these fields. We see that no field of the adjoint vector multiplet is involved in the construction of $\mathcal{C}^I$. A similar argument holds for the $\Delta = 2$ operator $\mathcal{O}^A$.

It is interesting to observe that the supergroup OSp(4/4) is the same as in the T-dual D3/D5 brane configuration. In the corresponding defect conformal field theory studied in [15, 16], the operators $\mathcal{C}^I$ and $\mathcal{O}^A$ have a similar structure and are interpreted as a chiral primary and one of its descendants. Due to different quantum numbers of the adjoint hypermultiplet in the defect conformal field theory, the trace terms are absent in the corresponding defect operators. This is due to the adjoint hypermultiplet being a four-dimensional bulk field in the defect CFT. For instance, the conformal dimension of the three-dimensional scalars ϕ_1, ϕ_2 is $\Delta = 1/2$, while in four dimensions, scalars have $\Delta = 1$. Another difference to the probe limit defect theory is the following: In the D3/D5 configuration the elements of a series of short representations of OSp(4/4) are given by the bosonic open string fluctuations on the D5-brane [15]. In our case these elements are provided by the supergravity fluctuations of the D2/D6 intersection.

3.2 Scalar meson spectrum

The derivation of the meson spectrum is very similar to the computation of the glueball spectrum in confining gauge/gravity dualities. The mass spectrum of the scalar glueball $J^{PC} = 0^{++}$ in pure QCD$_3$, for instance, is obtained [19, 20, 21, 22] by solving supergravity wave equations in the AdS black hole geometry [8]. The computation is based on the duality between the operator $\mathrm{Tr}\, F^2$ and the dilaton which satisfy the relation $m^2 = \Delta(\Delta - 4)$ of a massless scalar and a dimension four operator. In order to find the lowest glueball mass modes, one solves the equation of motion for the dilaton fluctuations by means of the plane wave ansatz [19]

$$\Phi = f(w) e^{ikx} , \tag{3.4}$$

where $f(w)$ is a function of the radial coordinate w in the black hole geometry. The glueball mass M is given by the eigenvalues of k^2 by $M^2 = -k^2$. Asymptotically the black hole geometry is AdS, where the dilaton has the behaviour $\Phi \sim 1, w^{-4}$ and only the latter is a normalizable solution. For fixed values of M, one then searches for normalizable solutions of the dilaton equation of motion with asymptotical behaviour w^{-4}.

For the meson spectrum we proceed in the same way using now the field-operator map derived in the previous section. This is a map between the operator $\mathcal{O}^A$ as given by eq. (3.3) and the supergravity field ϕ transforming in the $(\mathbf{3}, \mathbf{1})$ of the R-symmetry group. In the supergravity description we solve the classical equation of motion of the scalar field ϕ with mass squared $m^2 = -2$,

$$\frac{1}{\sqrt{g}} \partial_\mu \left[\sqrt{g} g^{\mu\nu} \partial_\nu\right] \phi + 2\phi = 0 , \tag{3.5}$$

on the eleven-dimensional background (2.5). We assume that the fluctuations have the plane wave form

$$\phi(w,x) = f(w) \sin kx \,. \tag{3.6}$$

The equation of motion for the fluctuations becomes

$$f''(w) + \frac{4}{w} f'(w) + \left(\frac{16 M^2 F}{w^4} + \frac{2 F^{1/3}}{w^2} \right) f(w) = 0 \,, \tag{3.7}$$

where we set the scale $L = 1$ and defined $' \equiv \partial_w$. Here $M^2 = -k^2$ is the meson mass and $F(w;m)$ is a function of the quark mass m as defined by eq. (2.9). For $w \to \infty$ the term including the meson mass can be neglected and the differential equation reduces to $f'' + \frac{4}{w} f' + \frac{2}{w^2} f = 0$. The solutions of (3.7) have the asymptotic behaviour w^{-1} and w^{-2}. This agrees with the asymptotic behaviour of a supergravity scalar dual to the $\Delta = 2$ operator $\mathcal{O}^A$,

$$f(w) \sim c w^{-\Delta} + m w^{\Delta - 3} \,. \tag{3.8}$$

According to the standard AdS/CFT prescription m is interpreted as a source of the operator and c as the vev of that operator.

We determine $M^2 = -k^2$ in dependence of the quark mass m by the shooting method. The quark mass enters the equation of motion via the function $F(w;m)$. For a fixed quark mass m we numerically integrate the equation of motion (3.7) with boundary conditions $f \sim w^{-2}$ at large w. Solutions of the equation must be regular at all values of w, such that the allowed M^2 solutions can be found by tuning to these regular forms.

Formally the meson mass depends also on the angles ψ, β and θ because of the function F. Since the metric is not symmetric in these angles, we face the problem of identifying the radial direction which corresponds to the energy scale in the field theory. Clearly, this is the direction which connects the singularity at $w = \beta = 0$ with the M2-branes. Other directions are not perpendicular to the M2-branes and cannot be related to an energy scale in the field theory.

This can be seen by considering the self-energy of a quark which is represented by a fundamental string stretched between the D2-branes and the boundary at infinity. Figure 4 shows two such strings, one for $\beta \neq \pi/2$ (string 1), the other for $\beta = \pi/2$ (string 3). For $\beta \neq \pi/2$ the fundamental string (string 1) is perpendicular to the D2-branes but not to the D6-branes. This string configuration is not stable and decays to a string perpendicular to the D6-branes (i.e. string 1 decays to string 2 in figure 4.) Since now the string ends on the D6-branes and not on the D2-branes, we cannot identify the string with a charge in the field theory anymore. The only stable string configuration is obtained for $\beta = \pi/2$ (string 3).

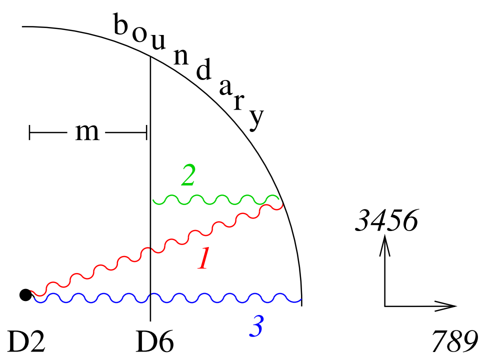

Figure 4: Self-energies of quarks represented by fundamental strings extending from the D2-branes and ending at infinity.

This string is perpendicular to both the D2 as well as the D6-branes and thus defines the appropriate radial direction. For more details on the relation between the self-energy of quarks and the radial direction in the geometry see also [17].

We thus choose the values for the angles as in eq. (2.8). This defines a radial direction in the w-β-plane as shown in figure 5. Unfortunately, for $y = \sin(\pi/2) \times \cos(0) = 1$, $F(w;m)$ diverges at $w = m$ which renders any meson computation impossible. We therefore regularize the function $F(w;m)$ either by approximating $F(w;m)$ by the step function $\Theta(w-m)$ or by deviating from $y = 1$. The latter approximation corresponds to a radial direction with $\beta \lesssim \pi/2$ which passes close to the M2-branes.

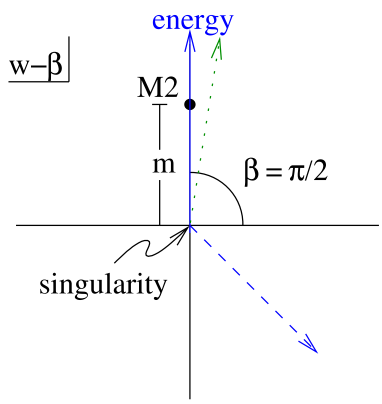

Figure 5: The radial direction corresponding to the energy scale in the field theory. The dashed direction does not have an energy interpretation. The dotted arrow represents the direction which we have chosen for the meson computation and corresponds to the regularization ($\beta \lesssim \pi/2$).

The Pelc-Siebelink metric possesses a coordinate singularity at the point $w = m$, $y = 1$, where the geometry reduces to an $AdS_4 \times S^7$ space. This point corresponds to the complete decoupling of quarks at very low energies. The regularization of F smoothes out this part of the metric making it suitable for the computation of meson spectra. This corresponds to a slight modification of the deep infrared region of the theory which should not affect the qualitative behaviour of the theory at energies above the quark mass.[10]

Figure 6 shows a plot of the meson mass in dependence of the quark mass for different values of y and for the step function approximation (labelled by Θ). As in the corresponding four-dimensional supersymmetric set-up [2] we find the meson mass to depend linearly on the quark mass. This is true for any of the regularizations we have considered although the slope of the graphs appears to be strongly regularization dependent. Nevertheless the linear behaviour is universal.

This behaviour of the meson mass agrees with expectations from field theory. We may see this by adapting a well-known argument using effective lagrangians. Consider an effective lagrangian for the meson-like field $\pi = \bar{\psi}^i \sigma^3_{ij} \psi^j + 2\bar{q}^\alpha \sigma q^\alpha$ of the type

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi \partial^\mu \pi - M_\pi^2 \pi^2\right) + \cdots . \tag{3.9}$$

The field π is essentially the third component of the operator $\mathcal{O}^A$ as defined in (3.3), where we ignored the terms with fields in the adjoint representation for simplicity. The meson mass M_π can be expanded in terms of the quark mass m as

$$M_\pi^2 = mB_0 + m^2 C_0 + \cdots , \tag{3.10}$$

where the coefficient B_0 is given by

$$B_0 = -\frac{\langle \bar{\psi}^i \sigma^3_{ij} \psi^j + 2\bar{q}^\alpha \sigma q^\alpha \rangle}{F_\pi^2} . \tag{3.11}$$

[10] A more rigorous way would be to find a different coordinate system in which the metric components are finite in the $AdS_4 \times S^7$ region of the Pelc-Siebelink background. We leave this for future investigations.

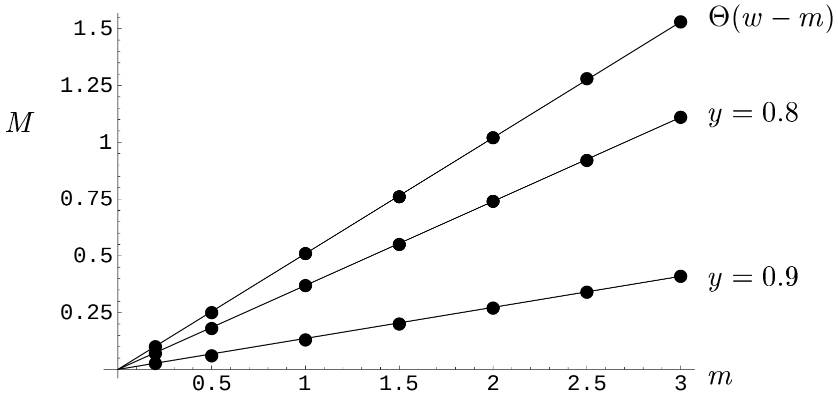

Figure 6: Lowest-lying meson mass as a function of the quark mass for three different regularizations. The meson mass scales linearly with the quark mass.

In a supersymmetric theory a quark condensate [1] or, in our case, a vev of the operator $\mathcal{O}^3$ is forbidden. The reason is that this operator is an F-Term of the mass operator $m\sigma^3_{ij}\bar{Q}^i Q^j$ and a vev for an F-term violates supersymmetry.[11] This argument holds in any dimension. We conclude that the meson mass M depends linearly on the quark mass m (at least for small quark masses). This is different from QCD in which there is no symmetry constraint to force $B_0 = 0$ leading to the behaviour $M_\pi \sim \sqrt{m}$ for small quark masses.

4. Conclusions

The results of this paper show that gauge/gravity duality with flavour may be consistently established beyond the probe approximation. As an example we have considered the holographic dual of a three-dimensional SU(N_c) super Yang-Mills theory with N_f fundamental fields in the limit $1 \ll N_f \ll N_c$ with N_f/N_c fixed and finite. We showed that meson-like operators may consistently be taken to be dual to closed strings. In this way we obtain the lowest-lying meson mass directly from supergravity fluctuations. This is new as compared to previous examples of gauge/gravity duals with flavour, in which meson operators are dual to open strings ending on a probe brane. The meson mass obtained from supergravity agrees with field theory expectations since it scales linearly with the quark mass.

It would be interesting to study further examples of fundamentals in backgrounds with backreaction. A similar analysis in four spacetime dimensions certainly deserves attention, perhaps by using the D3/D7 solution found in [13]. A four-dimensional version of the approach presented here would also allow for further insight into the holographic version of the Witten-Veneziano mass formula $M^2_{\eta'} \sim \nu\chi_T$ for the η' particle (see [11] for a stringy discussion in the probe limit). Keeping $\nu = N_f/N_c$ finite and fixed requires the backreaction to be taken into account. - In a somewhat different direction, it would be interesting to explore if gauge/gravity duality with flavour may be related to the work of [36] where the semiclassical string spectrum for the black hole background dual to pure QCD$_4$ is discussed.

[11] For the scalar contribution $\bar{q}^\alpha \sigma q^\alpha$ recall $\langle X^3_V \rangle \equiv \langle \sigma \rangle = m$ such that it contributes to B_0 rather than to C_0.

Another possible extension of this work is to consider mesons in a three-dimensional non-supersymmetric background. Ferretti et al. have shown [34] that there is a parity preserving phase of three-dimensional QCD in which the global $\mathrm{U}(2N_f)$ symmetry is broken spontaneously down to $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$. The dependence of the meson spectrum on the parameter ν is important: It has been found in [37] that the spontaneous symmetry breaking scenario does not take place for all values of $\nu = N_f/N_c$. There are some indications for the existence of a critical value $\nu^{\mathrm{crit}} = 64/3\pi^2 \approx 2.16$ (N_c large) above which the $\mathrm{U}(2N_f)$ flavour symmetry is restored and the theory does not exhibit dynamical symmetry breaking. It is a challenge to show this in terms of a dual holographic set-up.

Acknowledgments

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A. Field theory action

The field theory corresponding to the low-energy limit of the D2/D6 intersection is T-dual to the defect conformal field theory on the D3/D5 system studied extensively in [15, 16]. The world-volume theory is $\mathcal{N} = 4$ $\mathrm{SU}(N_c)$ super Yang-Mills theory coupled to N_c adjoint and N_f fundamental hypermultiplets. We will use a manifest $\mathcal{N} = 2$, $d = 3$ superspace formalism which can be obtained from $\mathcal{N} = 1, d = 4$ superspace upon dimensional reduction. Note that only a $\mathrm{U}(1)$ subgroup of $\mathrm{SU}(2)_V$ is manifest in $\mathcal{N} = 2, d = 3$ superspace language.

The field content of this theory is listed in Tab. 1. Under gauge transformations the $\mathcal{N} = 2$ superfields transform as

$$\begin{aligned} \Phi_i &\to e^{-i\Lambda}\Phi_i e^{i\Lambda}\,, \qquad & e^V &\to e^{-i\Lambda^\dagger} e^V e^{i\Lambda}\,, \\ Q_f &\to e^{-i\Lambda} Q_f\,, \qquad & \tilde{Q}_f &\to \tilde{Q}_f e^{i\Lambda}\,, \end{aligned} \tag{A.1}$$

where Λ is a chiral multiplet. As in $\mathcal{N} = 1, d = 4$ superspace we have three chiral and one vector superfield. The fields Q_f and $\tilde{Q}_f$ transform in the fundamental and antifundamental representation of the gauge group, respectively. We can also define a linear multiplet Σ by

$$\Sigma \equiv \epsilon_{\alpha\beta}\bar{D}_\alpha(e^{-V} D_\beta e^V) \tag{A.2}$$

which transforms under gauge transformations as

$$\Sigma \to e^{-i\Lambda}\Sigma e^{i\Lambda}\,. \tag{A.3}$$

In terms of these superfields the low-energy effective action of the D2/D6 intersection is given by

$$S = S_{\mathrm{D2}} + S_{\mathrm{quarks}}\,, \tag{A.4}$$

with the $\mathcal{N}=8$, $d=3$ super Yang-Mills action

$$S_{\mathrm{D2}} = \frac{1}{g^2}\int d^3x d^2\theta d^2\bar\theta \;\mathrm{Tr}\left[\frac{1}{4}\Sigma^2 - e^{-V}\bar\Phi_3 e^V \Phi_3\right] - \sum_{i=1}^{2}\int d^3x d^2\theta d^2\bar\theta \;\mathrm{Tr}\, e^{-V}\bar\Phi_i e^V\Phi_i + \\ + \int d^3x d^2\theta \;\mathrm{Tr}[\Phi_1,\Phi_2]\Phi_3 + \mathrm{c.c.} \tag{A.5}$$

and

$$S_{\mathrm{quarks}} = \int d^3x d^2\theta d^2\bar\theta\,(\bar Q_f e^V Q_f + \tilde Q_f e^{-V}\bar{\tilde Q}_f) + \left(\int d^3x d^2\theta\,\tilde Q_f \Phi_3 Q_f + \mathrm{c.c.}\right). \tag{A.6}$$

The action (A.6) describes the couplings to the fundamental hypermultiplets which break supersymmetry from $\mathcal{N}=8$ down to $\mathcal{N}=4$. Note that the conformal dimension of the coupling constant is $[g]=1/2$. In order to have correct conformal dimensions, we need the factor $1/g^2$ in front of the kinetic term of the vector multiplet.

B. Pelc-Siebelink background in 10 dimensions

The ten-dimensional Pelc-Siebelink background is given by [17]

$$\frac{ds^2}{l_s^2} = \frac{\sin\beta}{N_6}\left[\frac{w^2}{16(LF)^{1/2}}dx_{||}^2 + (LF)^{1/2}\left(\frac{dw^2}{w^2}+d\tilde\Omega_6^2\right)\right], \tag{B.1}$$

$$e^\phi = \left(\frac{\pi^2 N_2 F}{2N_6^5}\right)^{1/4}(\sin\beta)^{3/2}, \tag{B.2}$$

which asymptotes to a warped product geometry $AdS_4 \times_w X_6$. Here the AdS_4 space is fibered over a compact manifold X_6.

This geometry is a ten-dimensional supergravity solution only for a certain parameter regime of ν. There are two bounds on ν given by requiring both the dilaton as well as the curvature to be small. The curvature $\mathcal{R}\sim\sqrt{\nu}/\sin^3\beta$ ($\nu = N_6/N_2$) is small for $\nu \ll 1$. To impede a blow-up of the dilaton, N_2 and N_6 must satisfy $N_2^{1/5} \ll N_6$ (or, equivalently, $N_2^{-4/5} \ll \nu$). In the large N_2 limit the parameter ν takes therefore values in the regime $0 \ll \nu \ll 1$. For smaller values of ν the dilaton becomes very large and a better description is given by the eleven-dimensional metric (2.5).

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